

The crossover of deconfinement by tunnelling in FL model

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We have discussed the tunnelling effect in FL model. The tunnelling coefficient is derived in the field configuration space by calculating the transition amplitude using the path integral at SPA and the dilute instanton gas approximation. By studying the tunnelling effect between the two degenerating vacuums at the critical temperature and chemical potential, we find that the system could be deconfined by tunnelling, which will change the first order deconfinement phase transition to crossover. The $T - \mu$ phase diagram of deconfinement with both first order phase transition and crossover including CEP is presented.

PACS numbers: 25.75.Nq, 12.39.Ki, 11.10.Wx, 03.65.Xp

I. INTRODUCTION

It is generally believed that at sufficiently high temperatures and densities there is a QCD phase transition from normal nuclear matter to QGP [1, 2], in which there are theoretically two kinds of phase transitions: chiral phase transition and deconfinement phase transition. The relation between these two phase transitions is still a puzzle [3, 4]. However it seems that the two phase transitions happen coincidently [5]. For the chiral phase transition at finite temperature, the quark-antiquark condensate $\langle \bar{q}_R q_L \rangle$ serves as a good order parameter. The order of the phase transition is fairly studied [6–8]. From lattice calculations, for $N_f = 2$ massless quark flavors, it is a second order phase transition at high temperatures in the chiral limit, while for the non chiral limit it is a crossover. It is difficult to make the calculations at finite densities for the lattice, however at small densities one can still derive that the transition in the non chiral limit is the crossover. At high densities, the chiral phase transition has been studied by many effective models [9–11]. It is generally regarded that at high densities it is a first order phase transition. Thus in the $T - \mu$ phase plane from the first order phase transition at high densities to the crossover at high temperatures there exists a critical end point(CEP). For deconfinement phase transition, it has not a good order parameter except for infinite quark mass limit, at which the Polyakov loop serves as an order parameter [12, 13]. In the earlier studies of deconfinement, the bag models, such as MIT model [14], SLAC model [15] and Friedberg-Lee(FL) model [16], had been often used to investigate the confinement mechanics and the thermodynamics of deconfinement phase transition. With the temperatures and densities increased, the bag will be broken, which results as a first order phase transition of deconfinement. In recent studies the Polyakov loop has been combined into the chiral models, such as Nambu-Jona-Lasinio model [17, 18] and linear sigma model [19–21], which allows to investigate the deconfinement phase transition within the chiral models. Though the Polyakov loop is not a good order parameter, it still serves as an indicator of a transition towards deconfinement. As it is the first order phase transition of deconfinement, the transition will have latent heat. However, from the recent results of the lattice at high temperatures the deconfinement is also a crossover instead of a first order phase transition at the real physical quark mass [22].

As the quarks are confined in a nucleon, there is huge binding energy. From hadronic phase to quark phase, there should be enormous latent heat, which will result as a first order phase transition. If it is a crossover, the transformation between the hadronic phase and the quark phase will be smooth. Thus it is difficult to understand where the huge latent heat goes. From the theoretical point of view, what is the physical mechanism lying behind? In this paper we wish to use the FL bag model to make a tentative study of the crossover mechanism of the deconfinement.

The FL model has been widely discussed in past decays [23–25]. It has been very successful in describing phenomenologically the static properties of hadrons and their behaviors at low energy. The model consists of quark fields interacting with a phenomenological scalar field σ . The σ field is introduced to describe the complicated non-perturbative features of QCD vacuum. There are two vacuums in this model: one is the perturbative vacuum and the other is the non perturbative vacuum or physical vacuum. In the real world the physical vacuum is stable. The energy difference between the two vacuums defines the bag constant B . The quarks are confined in a soliton bag. It naturally gives a color confinement mechanism in QCD theory. The model has been also extended to finite temperatures and densities to study deconfinement phase transition [26–30]. In the previous studies, the deconfinement phase transition has been often discussed by analyzing the changing of the two vacuums. When the temperature or density increases, the energy difference between the two vacuums decreases. Until the two vacuums degenerate, the bag constant becomes to zero, and the deconfinement phase transition takes place, which results as a first order phase

transition.

However the tunnelling effect between the two degenerate vacuums are not considered in these studies. It has been indicated by T. D. Lee that it is an interesting problem to consider the tunnelling effect between the two degenerate vacuums in field configuration space which can eliminate the degenerate vacuums in a finite size microscopic system [31]. Here we think that the tunnelling effect between the two vacuums may be important in deconfinement, which could change the first order deconfinement phase transition to crossover. And the tunnelling between the degenerate vacuums could be possible physical mechanism for the crossover of the deconfinement.

The organization of this paper is as follows: in section 2 the FL model is introduced. The field equation of the sigma field is treated in homogeneous case. An effective lagrangian is obtained from which a transition amplitude is given in a form of the path integral. In section 3, the transition amplitude is evaluated in Euclidean space in stationary phase approximation (SPA) and dilute instanton gas approximation. The tunnelling amplitude is further derived. In section 4, the tunnelling probability is evaluated for the different effective potentials at different critical temperatures and chemical potentials in finite size system in field configuration space. The tunnelling effect is studied in the deconfinement phase transition. We indicate that the first order deconfinement phase transition may be replaced by a crossover through tunnelling effect. A $T - \mu$ phase diagram of deconfinement with both first order phase transition and crossover including CEP is presented. The last section is the summary.

II. THE EFFECTIVE LAGRANGIAN AND THE TRANSITION AMPLITUDE OF FL MODEL

We start from the Lagrangian of the FL model,

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - g\sigma)\psi + \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) - U(\sigma), \quad (1)$$

where

$$U(\sigma) = \frac{1}{2!}a\sigma^2 + \frac{1}{3!}b\sigma^3 + \frac{1}{4!}c\sigma^4 + B. \quad (2)$$

ψ represents the quark field, and σ denotes the phenomenological scalar field. a, b, c, g and B are the constants which are generally fitted in with producing the properties of hadrons appropriately at zero temperature. We shift the σ field as $\sigma \rightarrow \sigma + \sigma'$ where the new σ and σ' are the vacuum expectation value and the fluctuation of the σ field respectively. Then the lagrangian becomes

$$\mathcal{L}' = \bar{\psi}(i\gamma_\mu\partial^\mu - m_q)\psi + \frac{1}{2}(\partial_\mu\sigma')(\partial^\mu\sigma') - \frac{1}{2}m_\sigma^2\sigma'^2 - U(\sigma), \quad (3)$$

where

$$U(\sigma) = \frac{1}{2!}a\sigma^2 + \frac{1}{3!}b\sigma^3 + \frac{1}{4!}c\sigma^4 + B. \quad (4)$$

$m_q = g\sigma$ and $m_\sigma^2 = a + b\sigma + \frac{1}{2}c\sigma^2$ are the effective masses of the quark and σ fields respectively. The interactions associated with the fluctuation σ' , such as σ'^3, σ'^4 and $\bar{\psi}\sigma'\psi$, are neglected in the usual tree level approximation. At the tree level, one can obtain the field equations,

$$(i\gamma_\mu\partial^\mu - g\sigma)\psi = 0, \quad (5)$$

$$\partial_\mu\partial^\mu\sigma' + m_\sigma^2\sigma' = 0, \quad (6)$$

$$\partial_\mu\partial^\mu\sigma = -\left(\frac{\partial U}{\partial\sigma} + \frac{1}{2}(b + c\sigma)\langle\sigma'^2\rangle + g\langle\bar{\psi}\psi\rangle\right) \equiv -\frac{\partial V_{eff}(\sigma)}{\partial\sigma}, \quad (7)$$

where $\langle\sigma'^2\rangle$ and $\langle\bar{\psi}\psi\rangle$ denote the contributions of the thermal excitations of σ and quark fields respectively. The second equality of equation (7) means that we define a thermal effective potential. These thermal functions will be determined in the later section. Here our discussion will focus on the equation (7). σ is the classic field which is time and space dependent. There are two physical treatments of σ . If one treats the σ time independent and space dependent which means the static case, the equation (7) will be the usual soliton equation which is solved for soliton solutions. This case is mostly studied for the confinement mechanics and hadron properties in the previous

literature. If one treats the σ time dependent and space independent which means the homogeneous case. In this case the solution of the equation will be the instanton which is related to the tunnelling effect. Here we are interested in tunnelling effect, so we take the homogeneous case and the equation becomes

$$\frac{d^2\sigma}{dt^2} = -\frac{\partial V_{eff}(\sigma)}{\partial\sigma}. \quad (8)$$

We can regard the σ field as a “space coordinate” of a particle moving in one dimension in a potential $V_{eff}(\sigma)$. By this analog we can further obtain the effective Lagrangian density which describes the motion of the particle,

$$\mathcal{L}_{eff} = \frac{1}{2} \left(\frac{d\sigma}{dt} \right)^2 - V_{eff}(\sigma). \quad (9)$$

For the homogeneous case the effective Lagrangian is

$$L_{eff} = \int d^3x \mathcal{L}_{eff} = \Omega \left[\frac{1}{2} \left(\frac{d\sigma}{dt} \right)^2 - V_{eff}(\sigma) \right], \quad (10)$$

in which Ω is the space volume. The effective Lagrangian could be quantized by the canonical quantization. The corresponding quantized Hamiltonian in quantum theory is

$$H = \frac{\hat{p}^2}{2\Omega} + \Omega V_{eff}(\sigma), \quad (11)$$

where $\hat{p} = -i\hbar\partial/\partial\sigma$ is the canonical momentum operator. Consequently the quantum transition amplitude for a particle to start at “position” $\sigma = \sigma_a$ at time $t = 0$, and end up at $\sigma = \sigma_b$ at $t = T$ is given by

$$K(\sigma_b, T; \sigma_a, 0) = \langle \sigma_b | \exp(-iHT/\hbar) | \sigma_a \rangle. \quad (12)$$

By the path integral the transition amplitude can be also written as

$$K(\sigma_b, T; \sigma_a, 0) = \int D[\sigma(t)] \exp\{i/\hbar S[\sigma(t)]\}, \quad (13)$$

where

$$S[\sigma(t)] = \int_0^T dt L_{eff} = \int_0^T dt \Omega \left[\frac{1}{2} \left(\frac{d\sigma}{dt} \right)^2 - V_{eff}(\sigma) \right], \quad (14)$$

is the action and $\int D[\sigma(t)]$ is an integral over all path.

III. INSTANTON AND TUNNELLING EFFECT

In order to evaluate the transition amplitude, it is more convenient to do the calculation in a Euclidean space. The corresponding Euclidean transition amplitude denotes as

$$K_E(\sigma_b, \tau/2; \sigma_a, -\tau/2) = \int D_E[\sigma(\tau')] \exp[-1/\hbar S_E(\sigma(\tau'))], \quad (15)$$

where

$$S_E[\sigma(\tau')] = \int_{-\tau/2}^{\tau/2} d\tau' \Omega \left[\frac{1}{2} \left(\frac{d\sigma}{d\tau'} \right)^2 + V_{eff}(\sigma) \right] \quad (16)$$

is the Euclidean action, $\tau = iT$ and $\tau' = it$. One can also obtain the Euclidean equation of motion,

$$\frac{d^2\sigma}{d\tau'^2} = \frac{\partial V_{eff}(\sigma)}{\partial\sigma}. \quad (17)$$

In the following discussion we take $\tau \rightarrow \infty$ and we suppose $V_{eff}(\sigma)$ has the configuration schematically sketched in Fig.1. The potential has two minima which are degenerate. The equation (17) could be solved by the boundary

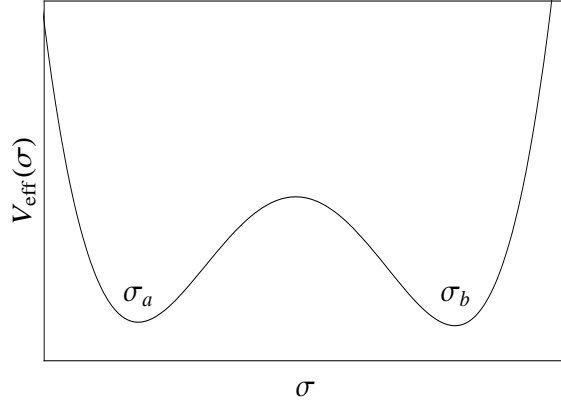


FIG. 1: The schematically sketched thermal effective potential as a function of σ .

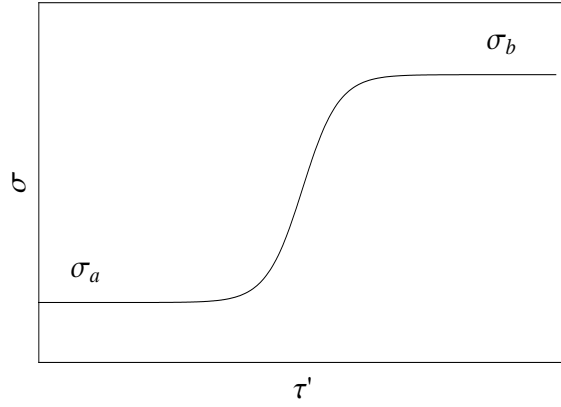


FIG. 2: A single instanton solution in the FL model.

condition

$$\tau' = -\infty, \sigma = \sigma_a; \quad \tau' = \infty, \sigma = \sigma_b. \quad (18)$$

The solution is an instanton which is schematically sketched in Fig.2. This instanton solution $\sigma_{cl}(\tau')$ is a classical path which is an extremum of the action $S_E[\sigma(\tau')]$. One could expand the action $S_E[\sigma(\tau')]$ about the classical path $\sigma = \sigma_{cl}(\tau')$,

$$S_E[\sigma(\tau')] = S_E[\sigma_{cl}(\tau')] + \frac{\Omega}{2} \int_{-\infty}^{\infty} d\tau' y(\tau') O_E(\tau') y(\tau') + O(y^3), \quad (19)$$

where

$$y(\tau') = \sigma(\tau') - \sigma_{cl}(\tau'), \quad (20)$$

and

$$O_E(\tau') = -\frac{\partial^2}{\partial \tau'^2} + \left(\frac{\partial^2 V_{eff}(\sigma)}{\partial \sigma^2} \right)_{\sigma_{cl}(\tau')}. \quad (21)$$

In the spirit of the stationary phase approximation (SPA), the major contribution to the transition amplitude (15) will come from the vicinity of the classical path $\sigma(\tau') = \sigma_{cl}(\tau')$. In that case, to leading approximation one can neglect the cubic and higher terms in $y(\tau')$ in the Taylor series (19). The SPA as applied to the path integral in the transition amplitude gives

$$K_E(\sigma_b, \tau/2; \sigma_a, -\tau/2) = e^{-S_0/\hbar} B_E(\tau) [Det O_E(\tau')]^{-\frac{1}{2}}, \quad (22)$$

where $S_0 = S_E[\sigma_{cl}(\tau')]$ and $B_E(\tau)$ is the measure factor in functional integral. Det is the determinant of the operator $O_E(\tau')$ which is most conveniently obtained from its eigenvalues, but the operator will have a zero eigenvalue because the action is translation invariant in the τ' variable. Formally this will cause a divergence in (22). However this zero mode could be separated from the determinant. For a detail discussion of this treatment one could refer to [32]. After separating the zero mode the result is

$$K_E(\sigma_b, \tau/2; \sigma_a, -\tau/2) = e^{-S_0/\hbar} J \tau B_E(\tau) [Det' O_E(\tau')]^{-\frac{1}{2}}, \quad (23)$$

where $J = \sqrt{S_0}$ and Det' denotes the determinant without the zero mode. In order to calculate this determinant, one could be referred to the case of harmonic oscillator. If the particle oscillates in the vicinity of the “position” σ_a , it could be regarded as a harmonic oscillator. So is it in the vicinity of the “position” σ_b . We suppose the angular frequency of the harmonic oscillator is ω . The determinant can be written as

$$[Det' O_E(\tau')]^{-\frac{1}{2}} = \left[Det \left(-\frac{d^2}{d\tau'^2} + \omega^2 \right) \right]^{-\frac{1}{2}} k, \quad (24)$$

where k is a constant independent of τ as $\tau \rightarrow \infty$. For the harmonic oscillator, the determinant could be evaluated at $\tau \rightarrow \infty$ and the result is

$$B_E(\tau) \left[Det \left(-\frac{d^2}{d\tau'^2} + \omega^2 \right) \right]^{-\frac{1}{2}} = \left(\frac{\omega}{\pi \hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega \tau}{2}}. \quad (25)$$

Hence the amplitude (23) becomes

$$K_E(\sigma_b, \tau/2; \sigma_a, -\tau/2) = e^{-S_0/\hbar} J k \tau \left(\frac{\omega}{\pi \hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega \tau}{2}}. \quad (26)$$

From the above derivation we obtain the result of the transition amplitude by considering single instanton. From the notion of tunnelling, this means the particle traverses the potential barrier once. However the particle could also traverse the potential barrier several times, that means there are also multi-instanton solution which configuration is shown in Fig.3 that will have the correct boundary condition for our problem, namely $\sigma(-\infty) = \sigma_a$, $\sigma(\infty) = \sigma_b$. The instantons should be widely separated and is so called the instanton gas. For our interest of tunnelling effect,

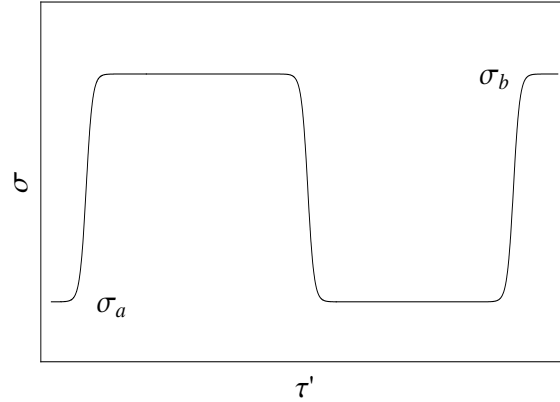


FIG. 3: Multi-instanton solution in the FL model.

the contributions from all these multi-instanton solutions to the transition amplitude should be summed up. The summation could be evaluated under the dilute instanton-gas approximation [32] and the result is

$$K_E(\sigma_b, \tau/2; \sigma_a, -\tau/2) = \left(\frac{\omega}{\pi \hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega \tau}{2}} \sinh \left(J k \tau e^{-S_0/\hbar} \right). \quad (27)$$

By equation (12) the transition amplitude can be also written as

$$\langle \sigma_b | \exp(-H\tau/\hbar) | \sigma_a \rangle = \left(\frac{\omega}{\pi \hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega \tau}{2}} \sinh \left(J k \tau e^{-S_0/\hbar} \right). \quad (28)$$

By the derivation in a similar way to the above derivation one can also obtain three kinds of transition amplitudes in the following,

$$\langle \sigma_a | \exp(-H\tau/\hbar) | \sigma_b \rangle = \left(\frac{\omega}{\pi\hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega\tau}{2}} \sinh \left(Jk\tau e^{-S_0/\hbar} \right), \quad (29)$$

$$\langle \sigma_a | \exp(-H\tau/\hbar) | \sigma_a \rangle = \left(\frac{\omega}{\pi\hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega\tau}{2}} \cosh \left(Jk\tau e^{-S_0/\hbar} \right), \quad (30)$$

$$\langle \sigma_b | \exp(-H\tau/\hbar) | \sigma_b \rangle = \left(\frac{\omega}{\pi\hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega\tau}{2}} \cosh \left(Jk\tau e^{-S_0/\hbar} \right). \quad (31)$$

If we define the states

$$|\sigma_{\pm}\rangle = \frac{1}{\sqrt{2}}(|\sigma_a\rangle \pm |\sigma_b\rangle), \quad (32)$$

then from equations (28—31) we have

$$\langle \sigma_{\pm} | \exp(-H\tau/\hbar) | \sigma_{\pm} \rangle = \left(\frac{\omega}{\pi\hbar} \right)^{\frac{1}{2}} e^{-\frac{\omega\tau}{2} \pm Jk\tau e^{-S_0/\hbar}}. \quad (33)$$

From the above expression one could read out the energy

$$E_{\pm} = \frac{1}{2}\hbar\omega \mp \hbar Jk e^{-S_0/\hbar}, \quad (34)$$

from which one could see that the states $|\sigma_{\pm}\rangle$ are the eigen-states of H with the eigenvalues E_{\pm} . The first term in equation (34) is the energy without tunnelling while the second term is the energy correction coming from the contribution of the tunnelling through the potential barrier. The factor $e^{-S_0/\hbar}$ reveals the exponential dependence on the “barrier strength”. Notice that

$$S_0 = S_E[\sigma_{cl}(\tau')] = \int_{-\infty}^{\infty} d\tau' \Omega \left[\frac{1}{2} \left(\frac{d\sigma}{d\tau'} \right)^2 + V_{eff}(\sigma) \right]_{\sigma_{cl}(\tau')}, \quad (35)$$

in which $\sigma_{cl}(\tau')$ is the classical path and will obey

$$\frac{1}{2} \left(\frac{d\sigma}{d\tau'} \right)^2 = V_{eff}(\sigma). \quad (36)$$

The above equation can be derived from the equation (17) by integration. Substituting equation (36) into equation (35) and changing the integration variable we obtain

$$S_0 = \int_{\sigma_a}^{\sigma_b} d\sigma \Omega \sqrt{2V_{eff}(\sigma)}. \quad (37)$$

This quantity represents the strength of the potential barrier between σ_a and σ_b . It is clear that the tunnelling amplitude is

$$M = \exp \left(-\frac{S_0}{\hbar} \right) = \exp \left[-\frac{\Omega}{\hbar} \int_{\sigma_a}^{\sigma_b} d\sigma \sqrt{2V_{eff}(\sigma)} \right]. \quad (38)$$

The result is in accord with conventional WKB treatments of quantum mechanics in the position configuration space, but one should notice that the result here is in the field configuration space.

IV. CROSSOVER OF DECONFINEMENT BY TUNNELLING

By the tunnelling amplitude one can further obtain the tunnelling probability

$$\eta = |M|^2 = \exp \left[-\frac{2\Omega}{\hbar} \int_{\sigma_a}^{\sigma_b} d\sigma \sqrt{2V_{eff}(\sigma)} \right]. \quad (39)$$

It is clear that the tunnelling probability is dependent on the potential $V_{eff}(\sigma)$ and the volume Ω . The effective potential V_{eff} should be determined in finite volume Ω . However first we will determine the effective potential in the infinite volume limit. Recall equation (7), the thermal excitations of σ and quark fields could be evaluated by standard method of finite temperature field theory and the results are

$$\langle \sigma'^2 \rangle = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{E_\sigma} \frac{1}{e^{\beta E_\sigma} - 1}, \quad (40)$$

$$\langle \bar{\psi} \psi \rangle = -\gamma \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{m_q}{E_q} \left(\frac{1}{e^{\beta(E_q - \mu)} + 1} + \frac{1}{e^{\beta(E_q + \mu)} + 1} \right), \quad (41)$$

in which β is the inverse temperature, μ is the chemical potential, and $\gamma = 2(\text{spin}) \times 2(\text{flavor}) \times 3(\text{color})$. $E_\sigma = \sqrt{\mathbf{p}^2 + m_\sigma^2}$ and $E_q = \sqrt{\mathbf{p}^2 + m_q^2}$. For the values of the model parameters a, b, c and g , there are different choices. We have taken one set of values as $a = 17.7 fm^{-2}$, $b = -1457.4 fm^{-1}$, $c = 20000$, $g = 12.16$, which have been often used in the literature [28]. The effective mass of σ field is fixed at $m_\sigma = 550 MeV$. One should notice that the second equality of equation (7) means that we define a thermal effective potential. By equation (40) and (41) the thermal effective potential in the infinite volume limit is

$$V_{eff}(\sigma) = U(\sigma) + \frac{1}{\beta} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \ln(1 - e^{-\beta E_\sigma}) - \frac{\gamma}{\beta} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[\ln(1 + e^{-\beta(E_q - \mu)}) + \ln(1 + e^{-\beta(E_q + \mu)}) \right]. \quad (42)$$

Next we will consider the finite volume effect. In the case of a finite system of linear size L the momentum integral in the effective potential will be replaced by a sum, which means

$$V_{eff}(\sigma) = U(\sigma) + \frac{1}{\beta L^3} \sum_{\mathbf{k}} \ln(1 - e^{-\beta E_k}) - \frac{\gamma}{\beta L^3} \sum_i \left[\ln(1 + e^{-\beta(E_i - \mu)}) + \ln(1 + e^{-\beta(E_i + \mu)}) \right]. \quad (43)$$

For a cubic system with the volume $\Omega = L^3$ one should consider the boundary conditions. In the paper [33], it is indicated that one is free to choose the periodic boundary condition (PBC) or the anti-periodic boundary condition (APC), which will lead to different discretization in the summation. In the limit of $L \rightarrow \infty$, the two different discretizations all go to the same continuum limit. However in the realistic evaluation we find that APC will be more consistent with the intrinsic statistic property of the fermion, while PBC is more proper for the boson. Thus we choose the PBC on the summation relating to the sigma field, while the APC on the summation relating to the quark field. That means in equation (43) for the summation in the second term the discretized momentum is $k_i = 2\pi m_i/L$, while for the summation in the third term the discretized momentum is $l_i = \pi(2n_i + 1)/L$, where m_i and n_i are integers $m_i, n_i = \dots, -2, -1, 0, 1, 2, \dots$ and i denotes for x, y, z . For E_k and E_l we have $E_k = \sqrt{\mathbf{k}^2 + m_\sigma^2}$ and $E_l = \sqrt{\mathbf{l}^2 + m_q^2}$, in which $\mathbf{k}^2 = k_x^2 + k_y^2 + k_z^2$ and $\mathbf{l}^2 = l_x^2 + l_y^2 + l_z^2$. To determine the system size L , we mention that the charge radius of a nucleon is about $r_c \approx 0.8 fm$. For a nucleon in a cubic box with linear size L , one can take the value of the size $L \approx 2r_c \approx 1.6 fm$ [34]. Then the summations in equation (43) could be numerical evaluated at the given temperature and chemical potential.

Now we are in a position to discuss the effective potential at different temperatures and chemical potentials in a finite size system. At zero temperature and chemical potential, the $V_{eff}(\sigma)$ is just the $U(\sigma)$, which has two minima at $\sigma_a = 0$ and $\sigma_b \neq 0$ corresponding to the two vacuums. The $\sigma_b \neq 0$ is the absolute minimum which is the physical vacuum. When the temperature or chemical potential increases, by evaluating the effective potential from equation (43) we find that the energy difference between the two vacuums decreases. Until the two vacuums degenerate that the bag constant B becomes to zero, the deconfinement phase transition takes place, which determines the critical temperature and the critical chemical potential. In Fig.4 we show the thermal effective potentials at different critical temperatures and critical chemical potentials in the finite size L . It is clear that the phase transition here is first order. However one should notice the tunnelling effect between the two degenerating vacuums. When the tunnelling takes place, the physical vacuum will be smoothly transformed to the perturbative vacuum without any energy changing, and there will be no latent heat as a result. The first order phase transition will be replaced by a smooth crossover. The tunnelling probability depends on the effective potential. From Fig.4 we can see the potential barriers become flatter and narrower from the top one to the bottom one, which will result a smaller tunnelling probability by a qualitative analysis. In order to make it more accurate we will evaluate the tunnelling probability.

In the calculation of tunnelling probability, we take $\hbar = 1$ and the system volume is $\Omega = L^3 \approx 4.1 fm^3$. From equation (42) we could evaluate the thermal effective potential at the critical temperature and chemical potential. By equation (39) the tunnelling probability could be further determined numerically. The tunnelling probabilities for different critical temperatures and chemical potentials are shown in Table I. From the table we could find that

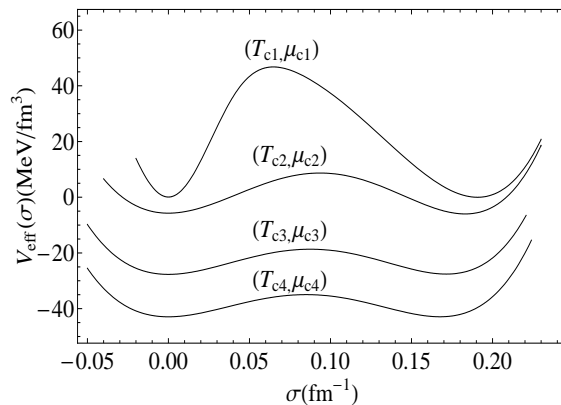


FIG. 4: The thermal effective potentials at the different critical temperatures and chemical potentials. From top to bottom the critical temperatures and chemical potentials are: $T_{c1} = 5\text{MeV}$, $\mu_{c1} = 668\text{MeV}$; $T_{c2} = 60\text{MeV}$, $\mu_{c2} = 480\text{MeV}$; $T_{c3} = 120\text{MeV}$, $\mu_{c3} = 239\text{MeV}$; $T_{c4} = 153\text{MeV}$, $\mu_{c4} = 0\text{MeV}$;

TABLE I: The tunnelling probabilities at the different critical temperatures and chemical potentials.

$T_c(\text{MeV})$	$\mu_c(\text{MeV})$	η
5	668	49.5%
20	628	56.0%
40	557	63.8%
60	480	68.1%
90	362	72.6%
120	239	75.1%
153	0	77.1%

the tunnelling probability increases with the critical temperature increasing (or with the critical chemical potential decreasing). That is to say at the high critical temperatures and the low critical chemical potentials along the phase boundary of deconfinement it is more probable for the tunnelling to occur than that at the low critical temperatures and the high critical chemical potentials. From Table I we can set a threshold value of tunnelling probability as

$$\eta_c \approx 75.1\%. \quad (44)$$

When $\eta > \eta_c$, we think that the tunnelling effect will dominate, and the transition from hadronic matter to quark matter is a crossover by tunnelling. When $\eta < \eta_c$, the tunnelling effect will be weaker, and the transition will be dominated by the first order phase transition. η_c is a critical point which value may not be the one we choose here, however there should exist a certain value of the critical point that separates the tunnelling process and the first order phase transition. This critical point is regarded as a critical end point of the first order phase transition. Therefore we can plot the $T - \mu$ phase diagram of deconfinement phase transition as shown in Fig.5. It is clear that the deconfinement phase transition is first order at the low critical temperatures and high critical chemical potentials while crossover at the high critical temperatures and low critical chemical potentials. From the first order transition to the crossover there exists a CEP.

V. SUMMARY

In this paper we have discussed the crossover of deconfinement by tunnelling in FL model. It is indicated that the system could be deconfined by tunnelling process which will lead to a crossover instead of a first order deconfinement phase transition. By the method of path integral we have derived the tunnelling amplitude in the field configuration space and obtain the tunnelling probability which is accord with the conventional WKB treatment of quantum mechanics. The tunnelling probabilities for the different thermal effective potentials in the finite volume in field configuration space at the different critical temperatures and chemical potentials are evaluated. We find that the tunnelling is more likely to take place at the high critical temperatures and low critical chemical potentials. A threshold value of tunnelling probability is set. Above that value the system is deconfined by tunnelling, in other

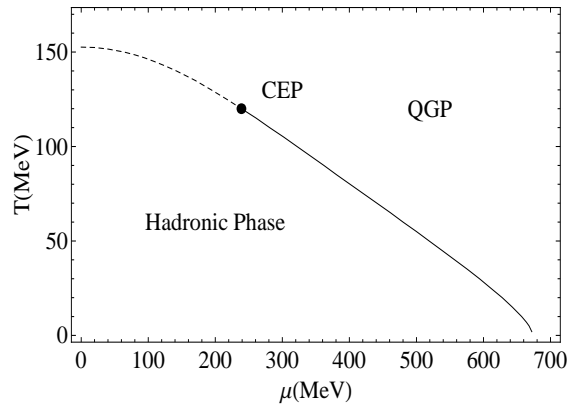


FIG. 5: $T - \mu$ phase diagram of deconfinement with CEP in the FL model.

words it is a crossover of deconfinement. Below that value it is a first order deconfinement phase transition. The $T - \mu$ phase diagram of deconfinement with both crossover and first order phase transition including the CEP is given. However it should be reminded that in this paper we only give one possible physical mechanism which could lead to the crossover of deconfinement, and the corresponding numerical results on the phase diagram are model dependent.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China with No. 10905018 and No. 10875050.

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